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Radiative transfer in
one-dimensional, discretely
stratified media

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Eugene A. Margerum

AUGUST 1980

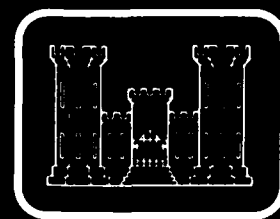
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PREFACE • This work was undertaken in connection with studies to develop theories and computational methods for scattering of radar waves by vegetation. To accomplish this, radiative transfer theory has been extended to layered media, where the scattering parameters vary from layer to layer and where the scattering parameters for any given layer can be computed from a knowledge of the distributions of vegetative scattering elements and appropriate theories for single scattering.

The extended theory eventually became a multi-channel (three-dimensional) formulation. The first steps in deriving this theory followed the historical precedent of deriving the formulas to treat the one-dimensional case, which is treated in the present report. Once this pattern was set, the equations were further extended to cover the multi-channel case, and these results will be given in a separate report.

In addition to setting a mathematical pattern to be followed in subsequent work, a case for diffuse reflection was developed from a semi-infinite homogeneous medium by using a mathematical limiting process. Since the results are of general interest, reflection coefficients were computed and tabulated for a wide range of values of the albedo for single scattering.

This work was performed in the Center for Theoretical and Applied Physical Sciences of the Research Institute of the U.S. Army Engineer Topographic Laboratories under work unit 4A161102B52C S3 0003.

COL Daniel L. Lycan, CE was Commander and Director and Mr. Robert P. Macchia was Technical Director of ETL during the report preparation.

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RADIATIVE TRANSFER IN ONE-DIMENSIONAL, DISCRETELY STRATIFIED MEDIA

INTRODUCTION • This research report discusses one-dimensional transfer of radiation in a medium composed of layers whose scattering properties vary from one level to the next. In addition, it is assumed that no radiation sources are present within the medium. The case considered is for diffuse reflection and transmission of externally incident radiation. The scattering properties of a single layer are assumed to be known, and the mathematical relations are developed for expressing the scattering properties of the multi-layer in terms of the scattering parameters of the single component layers.

This work was undertaken to determine radar scattering by vegetation. Since rigorous mathematical solutions are generally known only for homogeneous regions of relatively simple geometry, it was considered appropriate to use a discrete layer approach to account for the variation of scattering parameters with depth inside the vegetation. The radiation fields have been taken to be one-dimensional to establish the pattern of the mathematical solution. Subsequently, the solutions have been extended to radiation fields of higher dimensionality involving a discrete channel model. The results of the solutions will be given in separate reports, with the present report restricted to the simplified, one-dimensional case. This method of development is consistent with the historical precedent set by Schuster,¹ whose solution of a one-dimensional problem is generally considered to have laid the foundation for all further developments of radiative transfer theory.

In addition to establishing a framework upon which to build the discrete multi-channel theory, this research report also gives a new one-dimensional solution for back reflection from a homogeneous semi-infinite medium. To obtain this solution, an invariance principle is applied to the transfer equations in their discrete form. The transition to the case of a continuous scattering medium is then obtained by a mathematical limiting process. Since the resulting equation has a simple form, a table of backscattering functions has been computed and tabulated.

¹Schuster, Arthur, *Radiation Through a Foggy Atmosphere*, The Astrophysical Journal, Vol. 21, No. 1, January, 1905, pp. 1-22.

GENERAL FORMULATION • The scattering properties of a single component layer will be specified by its scattering coefficient σ and its transmission coefficient τ . In practice, these coefficients may be taken from

- a. Solutions of multiple scattering theories of radiative transfer theory
- b. Solutions of single scattering theories, if the layer is thin enough that the amount of multiple scattering within the layer is negligible
- c. Composite layers that have been assembled from layers of type a or b, using the methods to be presented below

The intensities of radiation flux in the forward and backward directions will be specified by I and R , respectively (figure 1).

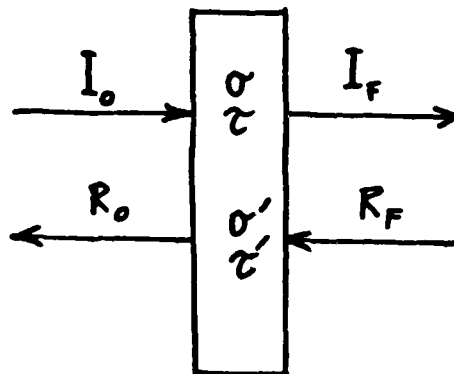


FIGURE 1. Single Component Layer Scattering Parameters.

In radiative transfer theory, it is customary to use the term "scattering (or reflection) function (or coefficient)" to specify a ratio of a backscattered intensity to an incident intensity. Similarly, the term "transmission function (or coefficient)" is used to denote the ratio of a transmitted intensity to an incident intensity. These coefficients will be defined more precisely below. In general it is necessary to use different coefficients for the scattering and transmission functions in opposite directions. However, for thin single-scattering layers, symmetry properties of the scatterers and reciprocity theorems for single scattering can sometimes be used to demonstrate that the same coefficients can be used for both forward and backward directions under certain conditions. Even when the reciprocity holds for single layers, it may fail for thicker multi-layers. This phenomenon is known as polarity. An example of its occurrence would be a double layer with one half purely absorbing and the other half purely scattering.

For the single component scattering layer being considered, the incident fluxes are I_o and R_F . The subscripts o and F denote that the radiation fields lie respectively to the left or right of the layer. In general, the scattering and transmission coefficients depend on whether the incident intensities are in the forward or backward direction. Thus, if $R_F = 0$, then σ and τ are defined by $\sigma = R_o/I_o$ and $\tau = I_F/I_o$. On the other hand, if $I_o = 0$, then $\sigma' = I_F/R_F$ and $\tau' = R_o/R_F$. In the general case, neither I_o nor R_F are assumed to be zero. Then, the emergent fields I_1 and R_o are given by the linear superposition of the contributions from both incident fields I_o and R_F .

$$I_1 = \tau I_o + \sigma' R_F \quad (1)$$

$$R_o = \sigma I_o + \tau' R_F \quad (2)$$

In the case of thin single-scattering layers, reciprocity theorems that apply to single scattering may be equivalent to the condition $\sigma = \sigma'$ and $\tau = \tau'$. It will be shown in the following section that when two different layers are combined, the corresponding coefficients for the composite layer will generally no longer satisfy such a reciprocity condition.

COMBINATION OF LAYERS • The relationship for scattering and transmission by two adjacent layers, whose individual scattering and transmission coefficients are given, will now be derived. The geometrical arrangement is shown in figure 2.

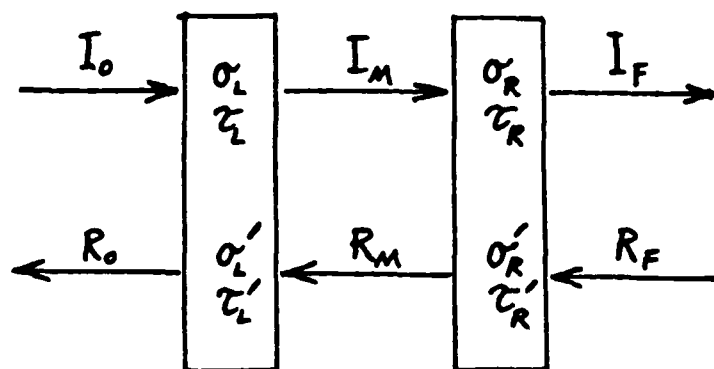


FIGURE 2. Combination of Two Component-Scattering Layers.

The intensities of the radiation fields between the layers, I_M and R_M , will contain not only transmitted radiation but also scattered radiation. Therefore, the possibility of multiple scattering between layers now arises, which will be reflected in the resulting equations.

If the left layer is considered by itself, the emergent intensities, I_M and R_0 , may be expressed in terms of the incident intensities, I_0 and R_M , in the manner given by equations 1 and 2.

$$I_M = \tau_L I_0 + \sigma'_L R_M \quad (3)$$

$$R_o = \sigma_L I_o + \tau'_L R_M \quad (4)$$

Similarly, the emergent intensities for the right layer, I_F and R_M , may be expressed in terms of its incident intensities I_M and R_F .

$$I_F = \tau_R I_M + \sigma'_R R_F \quad (5)$$

$$R_M = \sigma_R I_m + \tau'_R R_F \quad (6)$$

The immediate objective is to express the emergent intensities for the double layer, R_o and I_F , in terms of the incident intensities, I_o and R_F , by eliminating the intermediate intensities, I_M and R_M , from equations 1, 2, 3, and 4. This will be done by considering two special cases: $R_F = 0$ and $I_o = 0$.

Taking $R_F = 0$, equations 5 and 6 reduce to

$$I_F = \tau_R I_M \quad (7)$$

$$R_M = \sigma_R I_M \quad (8)$$

which, in turn, may be solved for I_M and R_M in terms of the intensity I_F .

$$I_M = \tau_R^{-1} I_F \quad (9)$$

$$R_M = \sigma_R \tau_R^{-1} I_F \quad (10)$$

When equations 9 and 10 are used to eliminate I_M and R_M from equation 3 after some algebraic manipulations, it is found that I_F can be expressed in terms of I_o .

$$I_F = \frac{\tau_R \tau_L}{1 - \sigma'_L \sigma_R} I_o \quad (11)$$

Also, when equations 9 and 11 are combined with equation 4 to eliminate R_M and I_F , it is possible to express R_o in terms of I_o .

$$R_o = \left(\sigma_L + \frac{\tau'_L \sigma_R \tau_L}{1 - \sigma'_L \sigma_R} \right) I_o \quad (12)$$

By an analogous procedure, if $I_o = 0$, so that the only source present is R_F , equations 3 and 4 become

$$I_M = \sigma'_L R_M \quad (13)$$

$$R_o = \tau'_L R_M \quad (14)$$

which can be solved for R_M and I_M in terms of R_o .

$$R_M = \tau'^{-1}_L R_o \quad (15)$$

$$I_M = \sigma'_L \tau'^{-1}_L R_o \quad (16)$$

By inserting equations 15 and 16 into equation 6 to eliminate R_M and I_M , an expression is found for R_o in terms of R_F .

$$R_o = \frac{\tau'_L \tau'_R}{1 - \sigma_R \sigma'_L} R_F \quad (17)$$

When equations 16 and 17 are combined with equation 5 to eliminate I_M and R_o , an expression for I_F in terms of R_F is found.

$$I_F = \left(\sigma'_R + \frac{\tau_R \sigma'_L \tau'_R}{1 - \sigma'_L \sigma_R} \right) R_F \quad (18)$$

The total solution, when I_o and R_F are both non-zero, is given by the superposition of the solutions given in equations 11, 12, 17 and 18.

$$I_F = \frac{\tau_R \tau_L}{1 - \sigma'_L \sigma_R} I_o + \left(\sigma'_R + \frac{\tau_R \sigma'_L \tau'_R}{1 - \sigma'_L \sigma_R} \right) R_F \quad (19)$$

$$R_o = \left(\sigma_L + \frac{\tau'_L \sigma_R \tau_L}{1 - \sigma'_L \sigma_R} \right) I_o + \frac{\tau'_L \tau'_R}{1 - \sigma_R \sigma'_L} R_F \quad (20)$$

Equations 19 and 20 give the emergent fields, I_F and R_o , for the double layer in terms of the incident fields I_o and R_F . By analogy, with the single layer case given by equations 1 and 2, it is possible to characterize the scattering properties of the composite double layer by reflection and transmission coefficients σ_C , σ'_C , τ_C , τ'_C .

$$I_F = \tau_C I_o + \sigma'_C R_F \quad (21)$$

$$R_o = \sigma_C I_o + \tau'_C R_F \quad (22)$$

$$\tau_C = \frac{\tau_R \tau_L}{1 - \sigma'_L \sigma_R} \quad (23)$$

$$\tau'_C = \frac{\tau'_L \tau'_R}{1 - \sigma_R \sigma'_L} \quad (24)$$

$$\sigma_C = \sigma_L + \frac{\tau'_L \sigma_R \tau_L}{1 - \sigma'_L \sigma_R} \quad (25)$$

$$\sigma'_C = \sigma'_R + \frac{\tau_R \sigma'_L \tau'_R}{1 - \sigma'_L \sigma_R} \quad (26)$$

Equations 23 through 26 show that unless the single component layers are identical and have reciprocal scattering properties with respect to direction, reciprocity does not hold for the composite double layer.

Clearly the formulas that have been developed in this section may be applied repetitively to build a scattering model for a medium consisting of many layers, if the scattering and transmission properties of each individual layer are known.

INTERPRETATION OF THE MULTIPLE SCATTERING TERMS •

Equations 23 through 26 contain fractional terms that account for the presence of multiple scattering. If these terms are expanded in a geometric series, each resulting term is found to represent a set of reflections and transmissions by/through the single component layers. This will be illustrated by taking equation 25 as an example. Using a geometric series transforms equation 25 to the following form:

$$\begin{aligned} \sigma_C = & \sigma_L + \tau'_L \sigma_R \tau_L + \tau'_L \sigma_R \sigma'_L \sigma_R \tau_L \\ & + \tau'_L \sigma_R \sigma'_L \sigma_R \sigma'_L \sigma_R \tau_L + \dots \end{aligned} \quad (27)$$

The first term accounts for radiation scattered once by the left layer. The second term accounts for radiation transmitted through the left layer, reflected by the right layer, and transmitted back through the left layer. In this manner, the factors comprising any given term account for the history of reflections and transmissions represented by each term. This is demonstrated schematically in figure 3.

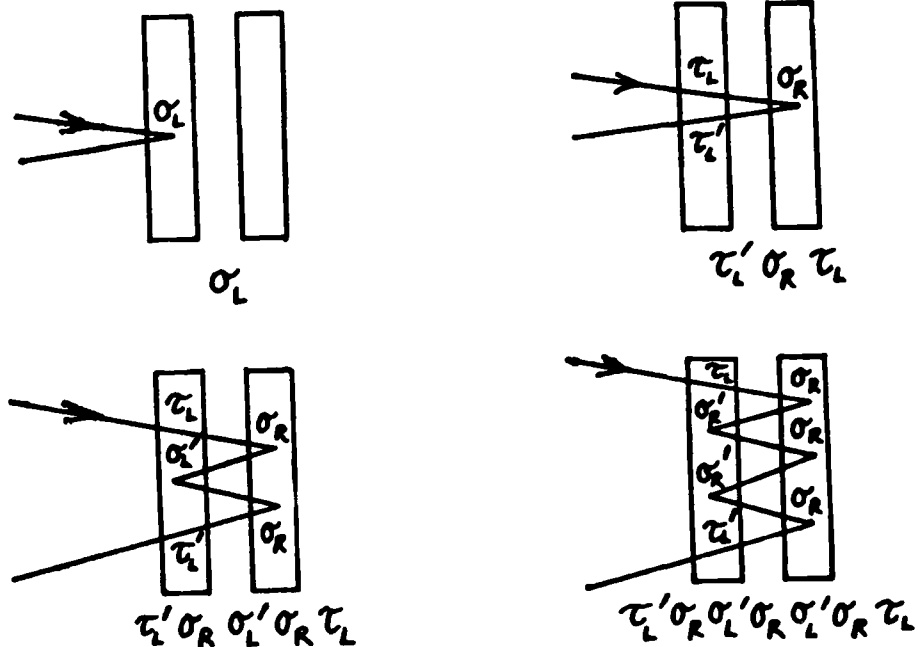


FIGURE 3. Interpretation of Terms in the Multiple Scattering Series.

In this way, an accounting is made for every possible combination of reflections and transmissions.

**THE SEMI-INFINITE HOMOGENEOUS CASE TREATED BY AN INVARI-
ANCE PRINCIPLE** • The reflection coefficient for a semi-infinite medium
composed of layers, all having the same scattering characteristics, can be
calculated by using an invariance principle. The medium will be described
by a notation similar to the one used in the preceding sections, but with a
new set of subscripts as shown in figure 4.

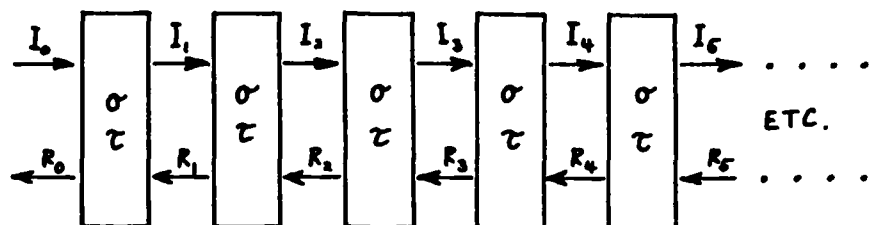


FIGURE 4. Reflection from a Semi-Infinite Stack of Scattering Layers.

The reflection coefficients for the complete array of scattering layers is given
by

$$R_0 = SI_0 \quad (28)$$

and the scattering and transmission coefficients, σ and τ , are the same for
each layer. Since the scattering array remains unchanged if the left layer is
removed, the condition of invariance can be stated as follows:

$$R_1 = SI_1 \quad (29)$$

The emergent intensities for the first plate, expressed in terms of the incident intensities, take the form

$$R_0 = \tau R_1 + \sigma I_0 \quad (30)$$

$$I_1 = \tau I_0 + \sigma R_1 \quad (31)$$

and the quantities R_0 and R_1 can be eliminated by means of equations 28 and 29.

$$SI_0 = \tau SI_1 + \sigma I_0 \quad (32)$$

$$I_1 = \tau I_0 + \sigma SI_1 \quad (33)$$

Both I_0 and I_1 can be removed from equations 32 and 33, resulting in a quadratic equation for S in terms of σ and τ .

$$\sigma S^2 + (\tau^2 - \sigma^2 - 1) S + \sigma = 0 \quad (34)$$

Equation 34 has a solution of the form

$$S = \frac{1 + \sigma^2 - \tau^2}{2\sigma} \pm \sqrt{\left(\frac{1 + \sigma^2 - \tau^2}{2\sigma}\right)^2 - 1} \quad (35)$$

where

$$1 - \sigma \geq \tau \quad (36)$$

and equality holds only if no absorption occurs. Squaring both sides of 36 (remembering that they are both positive) leads to the condition

$$\frac{1 + \sigma^2 - \tau^2}{2\sigma} \geq 1 \quad (37)$$

which combined with the fact that

$$S \leq 1 \quad (38)$$

indicates that the negative sign must be chosen in equation 35.

$$S = \chi - \sqrt{\chi^2 - 1} \quad (39)$$

$$\chi = \frac{1 + \sigma^2 - \tau^2}{2\sigma} \quad (40)$$

It is of interest to note that if equation 34 is written as

$$S = \sigma + \frac{\tau^2 S}{1 - \sigma S} \quad (41)$$

and the second term on the right is developed as a geometric series, an interpretation similar to that given in the preceding section can be made. Also, equation 41 can be used to develop an iteration scheme in which an approximate value for S , inserted on the right side, will yield a better approximation for S . These ideas take on more importance in a multi-channel formulation and will not be developed further in this report.

PASSAGE TO THE CONTINUUM AS A MATHEMATICAL LIMITING PROCESS •

The equations that have been developed up to this point are applicable to layers of finite thickness whose scattering properties are expressed by σ and τ . The single-scattering characteristics of the medium must be expressed ideally in terms of scattering parameters that pertain to an infinitesimal layer as its thickness approaches zero. Since in this limit both σ and τ always vanish, they cannot be used. Since it is customary to use the albedo for single scattering ϖ_0 , an analogous quantity will be defined for this one-dimensional case.

In conformity with Schuster,² it will be assumed that half of the scattered radiation will appear in the backward direction and half will appear in the transmitted flux. The fraction of radiation scattered is given by the scattering coefficient

$$s = 2\sigma \quad (42)$$

and the fraction removed from the incident flux by both scattering and absorption is given by the extinction coefficient

$$e = 1 - (\tau - \sigma) \quad (43)$$

where the term in parentheses accounts for the fact that the transmitted flux contains half of the scattered radiation. If the albedo for single scattering is defined by

$$\varpi_0 = \frac{s}{e} \quad (44)$$

it is then possible to express σ and τ as follows:

$$\sigma = \frac{\varpi_0 e}{2} \quad (45)$$

²Schuster, Arthur, *Radiation Through a Foggy Atmosphere*, The Astrophysical Journal, Vol. 21, No. 1, January, 1905, pp. 1-22.

$$\tau = 1 + \left(\frac{\varpi_0}{2} - 1 \right) e \quad (46)$$

Inserting these values into equation 40 yields:

$$\chi = \frac{2 - \varpi_0 - e(1 - \varpi_0)}{\varpi_0} \quad (47)$$

Furthermore, letting the layer thickness approach zero is mathematically equivalent to allowing the extinction coefficient to approach zero.

$$\begin{aligned} \tilde{\chi} &= \lim_{e \rightarrow 0} \chi \end{aligned} \quad (48)$$

$$\tilde{\chi} = \frac{2 - \varpi_0}{\varpi_0} \quad (49)$$

Insertion of this value into equation 39 results in a simple expression for the reflection coefficient \tilde{S} of a semi-infinite homogeneous medium whose scattering properties are specified entirely by

$$\tilde{S} = \frac{2 - \varpi_0 - 2\sqrt{1 - \varpi_0}}{\varpi_0} \quad (50)$$

or

$$\tilde{S} = \frac{(1 - \sqrt{1 - \varpi_0})^2}{\varpi_0} \quad (51)$$

In practice, σ_0 would be obtained from the values of s and e for a thin layer where the amount of multiple scattering would be very small.

TABULATED BACKSCATTERING FUNCTIONS FOR A HOMOGENEOUS SEMI-INFINITE MEDIUM • Reflection coefficients \tilde{S} have been computed for a wide range of values of the albedo for single scattering σ_0 by using equation 50. Since they may be of general interest, they are given below in table 1.

TABLE 1.
REFLECTION COEFFICIENTS $\tilde{S}(\varpi_0)$

0.0001	0.000025
0.001	0.00025
0.01	0.00251
0.05	0.01282
0.1	0.02633
0.2	0.05573
0.3	0.08893
0.4	0.1270
0.5	0.1716
0.6	0.2251
0.7	0.2922
0.8	0.3820
0.9	0.5195
0.95	0.6345
0.99	0.8182
0.999	0.9387
0.9999	0.9802
0.99999	0.9937
1.0	1.0000

CONCLUSION • In order to set the mathematical pattern of development of a suitable theory of radiative transfer for use in studies of radar scattering by vegetation, a theory of one-dimensional radiative transfer in discrete layered media was developed. By using this theory, it was shown how single layers with different scattering properties can be combined to obtain the scattering parameters for multi-layered media. An interpretation of the resulting equations was given that accounts for all possible multiple scattering between the component layers.

A semi-infinite homogeneous medium was also treated by using an invariance principle. In this case, the transition was made from the discrete to the continuous case, and a simple formula was given for the reflection coefficient in terms of the albedo for single scattering. Since the results are of general interest, a table covering a wide range of circumstances was prepared.

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